

“A grasshopper walks into a bar. The bartender says, ‘We’ve got a drink named after you.’ The grasshopper replies, ‘You’ve got a drink named Steve?’”

Unit Conversions—Dimensional Analysis

It is necessary to convert a measurement from one system of units to another, particularly for citizens and residents of the United States. In spite of the fact that all other countries of the world and all scientists use the metric system to express measured quantities, the U.S. still clings to an archaic British system of measurement, which even Great Britain no longer uses, having replaced it with the metric system.

For example, when your physician prescribes medication, he or she needs to convert your body weight to kilograms because dosages are usually expressed as milligrams of medication per kilogram of body weight. To convert a quantity from one system of units to another, medical personnel, scientists, and engineers frequently use a procedure called dimensional analysis.

Measured quantities are always represented by a number and its associated unit, such as 1.9 pounds or 3.5 inches. If you think of the number as a factor that multiplies the unit, you can apply standard algebraic conventions when you convert a measured quantity from one system of units to another. For example, to convert 3.45 kilograms to pounds, you multiply the given unit, kilograms, by a conversion factor that algebraically cancels the kilogram unit and yields pounds. Here’s the conversion:

$$3.45 \text{ kg} \times \frac{2.205 \text{ lb}}{1 \text{ kg}} = 7.61 \text{ lb}$$

Dimensional analysis works because the given unit is always multiplied by a conversion factor that is equal to one. The conversion factor comes from an equation that relates the given unit to the wanted, or desired, unit. For example, the equation

$$1 \text{ kg} = 2.205 \text{ lb}$$

defines the relationship between kilograms and pounds. If we divide both sides of this equation by 1 kg, we get a fraction that is equivalent to one:

$$\frac{1 \text{ kg}}{1 \text{ kg}} = 1 = \frac{2.205 \text{ lb}}{1 \text{ kg}}$$

The expression 2.205 lb/1 kg is a **conversion factor** that changes kilograms to pounds or vice versa. The “1 kg” quantity in this conversion factor is *exactly* 1 kilogram. Therefore when you use this conversion factor, the number of significant figures is determined by the number of significant figures in 2.205 lb.

Dimensional Analysis in Three Simple Steps

You can use the following three-step process to convert any given unit to the unit you need:

1. Write a conversion factor that relates the given unit to the wanted unit. If you cannot relate the two units directly with a single conversion factor, write a conversion factor that relates the given unit to an intermediate unit.
2. Multiply the given unit by the conversion factor from Step 1. Follow the algebraic rules from multiplication of fractions.
3. If the result of Step 2 is the wanted unit, the conversion is finished. If not, you must convert the intermediate unit. If another conversion is necessary, repeat Steps 1 and 2 until you arrive at the wanted unit.

Here is an example of a conversion that requires two steps:

Convert 3.00 feet to centimeters. 1 foot = 12 inches; 1 inch = 2.54 centimeters.

$$3.00 \text{ ft} \times \frac{12 \text{ in.}}{1 \text{ ft}} \times \frac{2.54 \text{ cm}}{1 \text{ in.}} = 91.4 \text{ cm}$$